

A Systematic Analysis of Supernova Light in Gamma-Ray Burst Afterglows

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ABSTRACT

We systematically reanalyzed all Gamma-Ray Burst (GRB) afterglow data published through the end of 2002, in an attempt to detect the predicted supernova light component and to gain statistical insight on its phenomenological properties. We fit the observed photometric light curves as the sum of an afterglow, an underlying host galaxy, and a supernova component. The latter is modeled using published multi-color light curves of SN 1998bw as a template. The total sample of afterglows with established redshifts contains 21 bursts (GRB 970228 - GRB 021211). For nine of these GRBs a weak supernova excess (scaled to SN 1998bw) was found, what makes this to one of the first samples of high- z core collapse supernovae. Among this sample are all bursts with redshifts less than ~ 0.7 . These results strongly support the notion that in fact all afterglows of long-duration GRBs contain light from an associated supernova. A statistics of the physical parameters of these GRB-supernovae shows that SN 1998bw was at the bright end of its class, while it was not special with respect to its light curve shape. Finally, we have searched for a potential correlation of the supernova luminosities with the properties of the corresponding bursts and optical afterglows, but we have not found such a relation.

Subject headings: gamma-rays: bursts – supernovae: general

1. Introduction

Observational and theoretical evidence suggest that the majority of GRB progenitors are stars at the endpoint in stellar evolution (e.g., Fryer, Woosley, & Hartmann 1999; Heger et

al. 2003). Since the discovery of a near-by Type Ic supernova (SN 1998bw) in the error circle of the X-ray afterglow for GRB 980425 (Galama et al. 1998; Kulkarni et al. 1998), evidence is accumulating that core collapse supernovae are physically related to long-duration GRBs. The supernova picture is further supported by the observation that all GRB hosts are star-forming, and in some cases even star-bursting galaxies (e.g., Frail et al. 2002; Sokolov et al. 2001). Evidence for host extinction by cosmic dust in GRB afterglows (e.g., Castro-Tirado et al. 1999; Klose et al. 2000), and the discovery of an ensemble of optically ‘dark bursts’ (for a recent discussion, see Fynbo et al. 2001; Klose et al. 2003; Lazzati, Covino, & Ghisellini 2002) also is consistent with the picture that GRB progenitors are young, massive stars (Groot et al. 1998; Paczyński 1998). Furthermore, for several GRB afterglows X-ray data suggest a period of nucleosynthesis preceding or accompanying the burst (e.g., Antonelli et al. 2000; Lazzati, Campana, & Ghisellini 1999; Mészáros & Rees 2001). The angular distribution of the afterglows with respect to their hosts also favors a physical relation of young, massive stars to GRBs (Bloom, Kulkarni, & Djorgovski 2002).

As a natural consequence of a physical relation between the explosion of massive stars and GRBs supernova light should contribute to the afterglow flux, and even dominate under favorable conditions. The most convincing example is GRB 030329 (Peterson & Price 2003) at $z=0.1685$ (Greiner et al. 2003a) with spectral confirmation of supernova light in its afterglow (Hjorth et al. 2003; Kawabata et al. 2003; Matheson et al. 2003; Stanek et al. 2003). In contrast to this unique spectroscopic evidence, several cases of photometric evidence for extra light in GRB afterglows have been reported, starting with GRB 980326 (Bloom et al. 1999). Various groups successfully fit SN 1998bw templates to explain these late-time bumps, the most convincing case being that of GRB 011121 (Bloom et al. 2002; Garnavich et al. 2003; Greiner et al. 2003b).

The goal of the present paper is to analyze the supernova bumps in GRB afterglow light curves using a systematic approach. While this was done also for several bursts by Dado, Dar, & de Rújula (2002a; and references therein) with respect to a verification of their cannonball model (Dar & de Rújula 2003), we tackle this issue in an independent and different way. First, from the numerical site, we have developed our own computational approach. This includes a numerical procedure to redshift SN 1998bw light curves (see § A.2) and to fit afterglow data within the context of the fireball model. Second, from the observational site, when necessary and possible we have performed late-time observations of some GRB host galaxies (§ 2). A considerable part of the data we have included in our study is based on observing runs where we have been involved. Additional data have been collected from the literature and checked for photometric consistency. Nearly two dozen afterglows had sufficient data quality, and a known redshift, to search for a late-time bump in the light curve (§ 3). Third, we concentrate our attention on a statistical analysis of the phenomenological parameters for

this class of GRB-SNe (§ 4). In this respect our investigation goes beyond the approaches undertaken by others to explain late-time bumps in individual afterglow light curves.

2. Observations and data processing

Some of the GRB afterglows we analyzed had poorly sampled late-time data, which made it difficult to find or determine the parameters of a SN bump (GRBs 000418, 991208, and 010921). In order to perform late-time photometry of these GRB hosts, we carried out two observing runs at the Calar Alto 3.5-m telescope on March 13–14 and May 23–25, 2003. Observations were performed using the multi-purpose camera *MOSCA*, which uses a SITe $2k \times 4k$ CCD with a plate scale of 0.32 arcsec per pixel. The field of view is 11×11 arcmin². During the first observing run the seeing varied between 1.4 and 1.6 arcsec; in May the seeing was better than 0.8 arcsec. Data reduction followed standard procedures.

Most of the light curves we investigated have been followed in more than one photometric band. For each of the GRBs we chose the best-sampled light curve as a reference light curve for the fit in the other photometric bands. In the most cases this was the *R* band light curve. In brief, our approach was as follows. *First*, we assumed that the afterglow slopes and break time are the same in all filters (Eq. A2), in reasonable agreement with observational data. Consequently, once we fit the reference light curve of an optical transient and deduced the corresponding afterglow parameters, we treated them as fixed parameters when fitting the light curves of the optical transient in other photometric bands. Thereby the fit procedure was based on a χ^2 -minimization with a Levenberg - Marquardt iteration. *Second*, for the representation of the supernova component we used published *UBVRI* data of the light curve of SN 1998bw (Galama et al. 1998) as a template, and redshifted them to the corresponding cosmological distance of the burster (§ A.2). These light curves are different from band to band. In addition, we allowed a variation of the SN luminosity with respect to SN 1998bw and a stretch in time (Eq. A1 in § A.1). *Third*, the host magnitude, which represents a constant component in the integral light of the optical transient, was usually considered as a free parameter. Only for GRB 011121 and 020405 we used host-subtracted magnitudes to fit the light curves.

Before performing a numerical fit, the observational data was corrected for Galactic extinction along the line of sight using the maps of Schlegel, Finkbeiner, & Davis (1998). This also holds for SN 1998bw, where we assumed $E(B - V) = 0.06$ mag. We calculated the Galactic visual extinction according to $A_V^{\text{Gal}} = 3.1 E(B - V)$, whereas the extinction in *U* and *B* were obtained via Rieke & Lebofsky (1985), and in *R_c* and *I_c* by means of the numerical functions compiled by Reichart (2001).

3. Results

Among the 36 ± 1 GRBs with detected optical afterglows up to the end of 2002¹, 21 had sufficient data quality, and a known redshift, for a meaningful search for an underlying supernova component (Table 1). Among them in nine cases evidence for a late-time bump was found. The results are summarized in Table 2. A general inspection of this table makes clear that the burst ensemble with detected late-time bumps in their afterglows separates into a group with statistically significant evidence for a bump (990712, 991208, 011121, and 020405), mostly in at least two photometric bands, and a group with less significant bumps (970228, 980703, 000911, 010921, and 021211). Given the fact that evidence for these bumps has also been reported by other groups (with the only exception being GRB 010921), we feel confident that the results presented in Table 2 can be used for a first statistical approach to understand this type of GRB-SNe.

The most interesting result is that our numerical procedure found evidence for a late-time bump in *all* GRB afterglows with a measured redshift $z \lesssim 0.7$. We believe that the interpretation of these bumps as an underlying supernova component is the most natural and observationally most founded explanation. Among the higher redshifted bursts, we confirm the finding by Holland et al. (2001) of a possible bump in the afterglow of GRB 980703, the discovery by Lazzati et al. (2001) of a bump in the afterglow of GRB 000911 ($z=1.06$; Price et al. 2002b), and a bump in the afterglow of GRB 021211, which was discovered by Della Valle et al. (2003) and is also discussed by Dado, Dar, & de Rújula (2003b).

For five afterglows (GRB 970508, 991216, 000418, 010222, 020813) with $0.7 < z < 1.5$, we can only place upper limits on the luminosity of any underlying supernova component. These five upper limits have typical uncertainties of a factor of two. The only exception is GRB 020813 where the uncertainty is much larger, so that no firm conclusions can be drawn here. The remaining seven bursts in our sample (GRB 971214, 990123, 990510, 000301, 000926, 011211, 021004) have redshifts $z > 1.5$ and, therefore, have not been investigated here, since this would have required a substantial extrapolation of the SN 1998bw template into the UV domain. Finally, the late-time bump clearly seen in the afterglow of GRB 980326 (Bloom et al. 1999) is not included in our study, because the redshift of the burster is not yet known.

As we have outlined in the previous section, we fit the SN component using the light curves of SN 1998bw as a template. Thereby, we allowed the luminosity and the light curve shape to be a free parameter. The former means a scaling of the luminosity of SN 1998bw

¹<http://www.mpe.mpg.de/~jcg/grbgen.html>

by a factor k (Eq. A1), whereby k always refers to the corresponding wavelength region in the redshifted SN frame. Differences in the light curve shapes were modeled by means of a stretch factor s , which allows the supernova light curve to develop slower ($s > 1$) or faster ($s < 1$) than the one of SN 1998bw (Eq. A1). In this respect we follow Hjorth et al. (2003) to describe the light curve of GRB 030329/SN 2003dh. The advantage of such an approach is that we can use these two parameters to explore the entire ensemble of GRB-SNe in a statistical sense. Table 2 shows that for the bursts with the photometrically best sampled late-time bumps in their optical light curves (GRB 011121, 020405) the deduced parameters k and s are consistent with each other in different wavelength regions. A comparison of these parameters of the nine afterglows with late-time bumps, which are at different redshifts, seems to be a reasonable first approach in order to constrain the width of the luminosity distribution of GRB-SNe.

In Fig. 1 we display the deduced parameter k (luminosity) for every individual GRB-SNe. We plot luminosity versus redshift just to look for a potential evolutionary effect (which is not apparent) and to separate the individual SNe from each other. While in Fig. 1a we allowed the stretch factor s to be a free parameter during the numerical fit, Fig. 1b shows the results obtained when we fixed $s = 1$. The reason for the latter was twofold. First, if $s = 1$ we can constrain the luminosity of an underlying SN for those GRBs, where we do not detect a bump in the late-time light curve. This is not possible in a reasonable way, if we allow s to be a free parameter. Second, sometimes the data base is too small in order to include also the stretch parameter in the fitting procedure, so that we have to fix s . Note that in Fig. 1 there are three small sets of bursts at redshifts 0.4, 0.7 and 1.0. Between them is no difference apparent neither in the luminosities of the GRB-SNe nor in the width of the luminosity distribution. What is apparent from a comparison of Fig. 1a,b is that introducing the stretch factor reduces the width of the luminosity distribution of the GRB-SNe and brings the luminosity of all SNe a little closer to those of SN 1998bw ($k = 1$). The distribution of the deduced stretch factor itself is shown in Fig. 2. Although s varies by a factor of two in both directions around $s = 1$, within their individual 1σ error bars most data are close to $s = 1$. Finally, no correlation was found between the deduced SN luminosity (parameter k) and the stretch factor s (Fig. 3).

4. Discussion

4.1. The luminosity distribution of the GRB-SNe

When plotting the parameter k deduced for the R band in the observer frame, the width of the luminosity distribution of the class of GRB-SNe (Fig. 4) is similar to what is

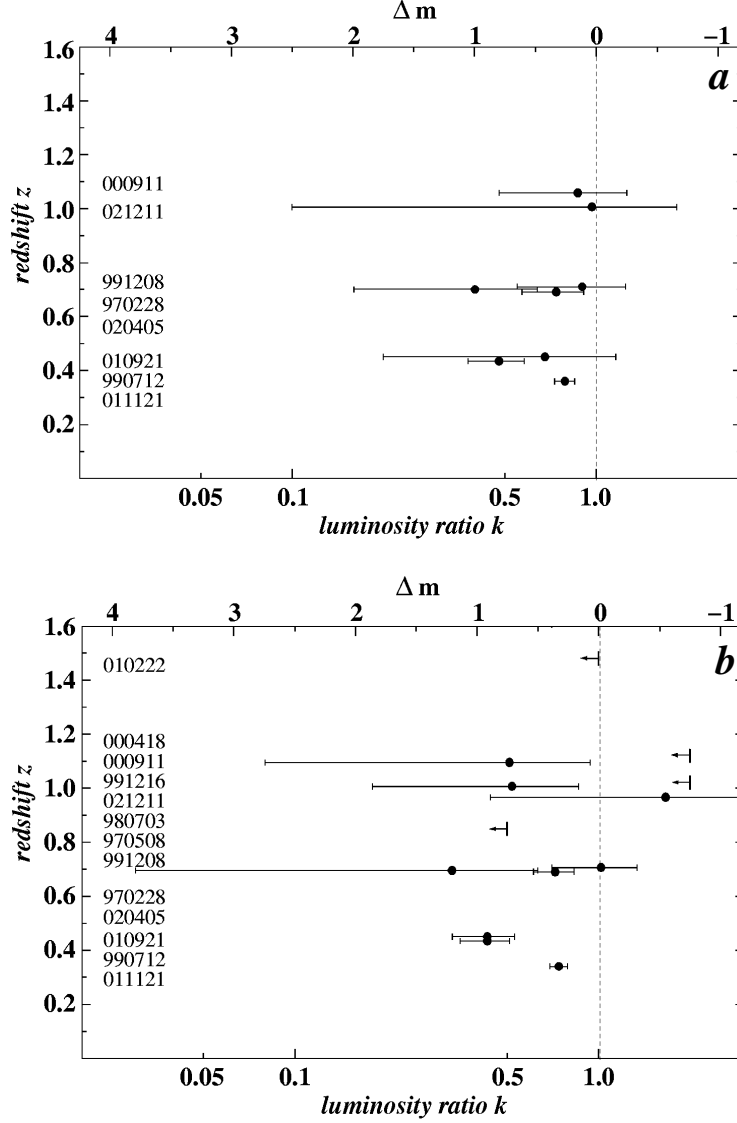


Fig. 1.— The deduced luminosities of GRB-SNe in units of the luminosity of SN 1998bw in the same spectral region (parameter k , Eq. A1). All data are based on observations in the R band. The broken line corresponds to SN1998bw; it is $\Delta m = -2.5 \log k$, which measures the magnitude difference at maximum light between the GRB-SN and SN 1998bw in the corresponding wavelength regime. The lower panel (b) is for a stretch parameter fixed at $s=1$, while in the upper panel (a) s is not fixed. In the former case we can set upper limits on k for further four bursts (GRB 970508, 991216, 000418, 010222). Moreover, the numerical procedure can fit the afterglow light curve of GRB 980703. Note that the data are not corrected for a possible extinction in the GRB host galaxies.

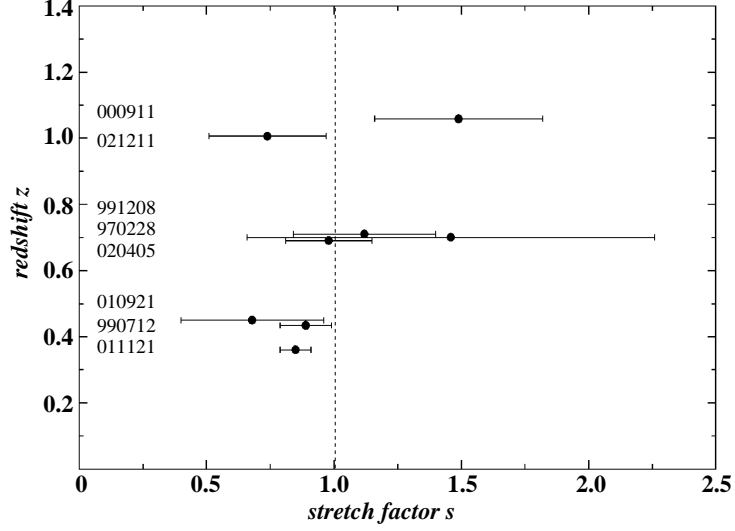


Fig. 2.— The distribution of the parameter s (Eq. A1) describing a stretching of the SN light curve relative to those of SN 1998bw (for which by definition $s = 1$, broken line). The mean value of s is close to 1.0.

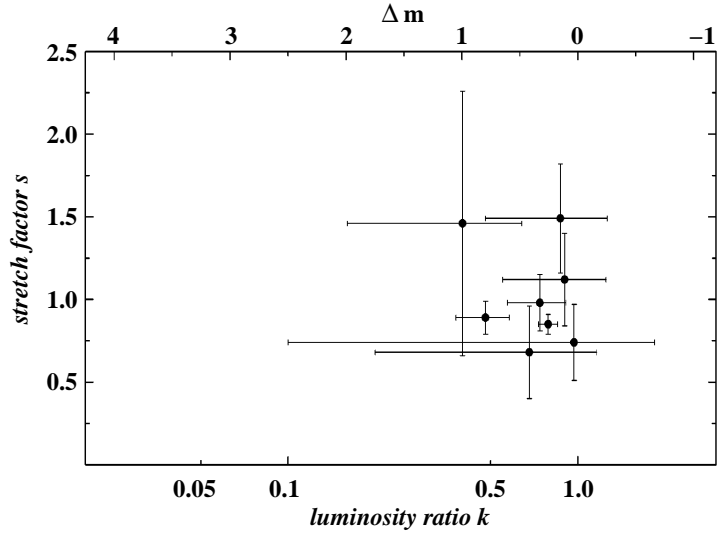


Fig. 3.— Drawn here is the luminosity ratio k versus the stretch factor s for the eight GRB-SNe of Figs. 1a and 2. No correlation between k and s is apparent in the data.

observed for other classes of core collapse SNe (Richardson et al. 2002). The mean of k in the R band is 0.7 independent of whether or not we fix the stretch parameter at $s = 1$, while our template SN 1998bw is at the bright end of the GRB-SN luminosity distribution. The latter conclusion is supported when we plot k for the same wavelength region in the SN host frame, which is a better indicator of the spread in SN luminosities. Most of the GRB-SNe we explored have a data point for the photometric band centered around 395 ± 10 nm in the corresponding host frame (Table 2). The distribution of k now indicates again that SN 1998bw is among the most luminous GRB-SNe. It also indicates that in fact there is no peak around $k=0.8$ (Fig. 4), but we may so far have only sampled the bright part of the GRB-SN luminosity function. Some caution is of course required, given the partly large error bars of the k factors, which are not shown in Figs. 4 and 5. While the conclusion that SN 1998bw was among the most luminous members of its class seems to be robust, the shape of the GRB-SNe luminosity function is still less well-determined. Extinction by interstellar dust in the host galaxies could in principle also affect these results, although only for GRB 010921 was a significant host extinction ($\gtrsim 1$ mag) reported (Price et al. 2003).

As we have outlined before, for redshifts $z \lesssim 0.7$ all GRB afterglows show evidence for an underlying late-time bump. Within our context this means that we trace a complete set of GRB-SNe, i.e., not only the brightest members of this class. The width of this GRB-SNe luminosity distribution in the photometric band centered around 395 ± 10 nm in the SN frame is ~ 1 to 1.5 magnitudes. This wavelength region is roughly placed in the B band, so that we can compare the corresponding luminosities with those of other Type Ib/c supernovae (Richardson et al. 2002), i.e., those class of SNe, which is believed to include the progenitors of GRBs. It turns out that the GRB-SNe do fit into a region between approximately $M_B = -19.5$ and $M_B = -18$ in figure 6 of Richardson et al., where no data on Type Ib/c SNe are known. If all GRB-SNe are indeed of type Ib/c this would favor the conclusion that the luminosity function of Type Ib/c SNe is rather described by a broad Gaussian than by a bimodal distribution (figures 6 and 7 in Richardson et al.).

The non-detection of a supernova bump in more than half of the investigated GRB afterglow light curves may be accounted for by several reasons, like a relatively bright host, or a faint supernova. In particular, finding a SN bump for high- z bursts is an observational challenge. For $z \gtrsim 0.7$ and $k = 1$, this peak magnitude exceeds $R_c = 24$, which poses a major challenge for 3-m class telescopes, given the usually very limited amount of target of opportunity time for such observations. It is thus no surprise that only for three of the GRBs above $z = 0.7$ a supernova component was found (GRB 980703, GRB 000911, GRB 021211), even though we can not rule out that the SN 'drop-out' is due to some evolutionary effect of the underlying burster population and their environment.

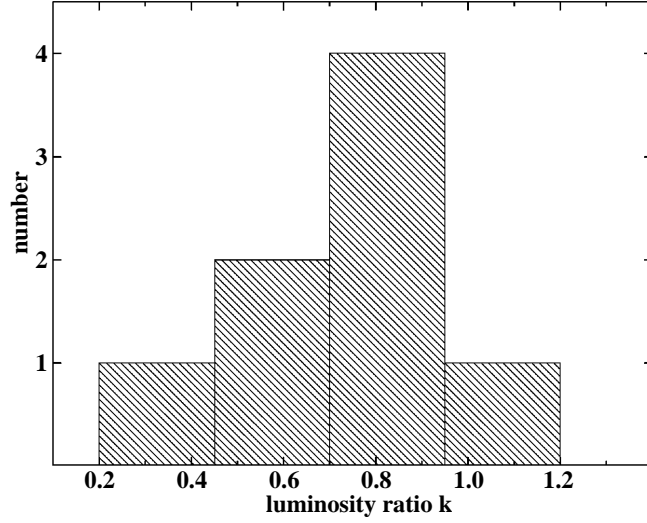


Fig. 4.— The distribution of the luminosity parameter k (Eq. A1) as measured in the R band in the observer frame (Table 2, with s being a free parameter). GRB 980703 is not included here because the fitting procedure did not find a solution in this case. Note that the histogram does not include the 1σ error bars of the individual k factors, which are relatively large.

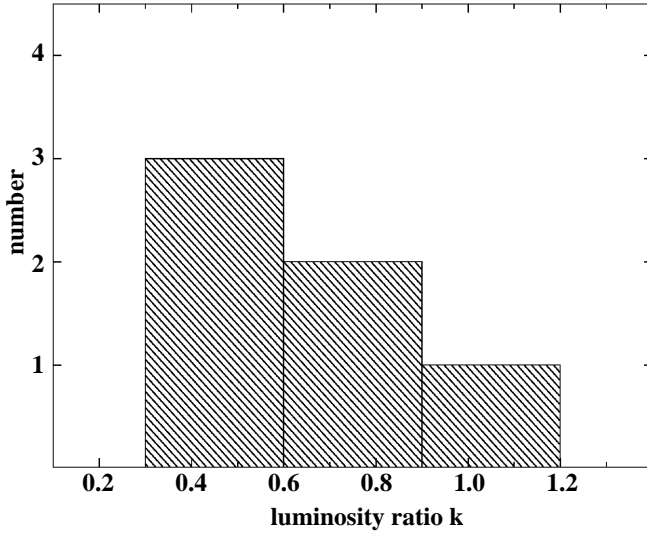


Fig. 5.— The same as Fig. 4, but for the photometric band centered around 395 ± 10 nm in the SN host frame (Table 2, with s being a free parameter). Not included here are GRB 980703, 010921, and 021211 since there is no data point in this wavelength range.

4.2. The SupraNova model

Vietri & Stella (2000) argued that GRBs are the result of delayed black hole formation, which implies that the core collapse and its subsequent supernova may significantly precede the burst. The delay could be of order months to years (Vietri & Stella 2000), or perhaps as short as hours (Woosley, Zhang, & Heger 2002). While constraining the latter possibility can not be accomplished with the data at hand, the longer time scales are easily constrained. For only two of the SN light curves the fit indeed improved if we allowed for a shift in time between the onset of the burst and the onset of the SN (GRB 990712, 011121). The offsets never exceeded 5 days, and were both negative and positive. However, the uncertainties in this parameter are large, due to the poorly sampled shape of the underlying supernova (e.g., Garnavich et al. 2003). The average shift is basically consistent with zero. Presumably, these shifts are due to an underlying correlation between luminosity and light curve shape, as observed in other types of supernovae (e.g., Candia et al. 2003; Stritzinger et al. 2002). This is just what the parameter s takes into account. On the other hand, it is clear that we have no information about this issue in those cases where we have not found evidence for a SN bump at late times. While this still leaves open the possibility of two populations of bursters (collapsars & SupraNovae), we emphasize again that we find a late time bump in all afterglows with redshift $z < 0.7$.

4.3. X-ray lines and supernova bumps

The identification of late-time bumps in afterglow light curves with SN light would benefit from observations in the X-ray band (for the cannonball model, e.g., Dado, Dar, & de Rújula 2003a). If the X-ray lines seen in some afterglows (e.g., Reeves et al. 2002) have their origin in the circumburst medium (e.g., Lazzati et al. 2001) and not in the exploding star (e.g., Mészáros & Rees 2001) this would be difficult to reconcile with the interpretation of a late-time bump with an underlying SN component. Unfortunately, the majority of bursts with high-resolution *XMM-Newton* or *Chandra* spectroscopic X-ray follow-up observations have no well-observed optical light curves. Among the afterglows with a detected optical late-time bump listed in Table 2 only for GRB 020405 such observations exist (Mirabal, Paerels, & Halpern 2003); no evidence for X-ray lines has been found there. *BeppoSAX* observed the afterglow of GRB 970228 (Frontera et al. 1998) with comparable low energy resolution, and no X-ray lines have been reported. Among those low- z bursts with well-defined optical light curves and no evidence for a late-time bump in the data, only for GRB 970508 *BeppoSAX* X-ray follow-up observations have been published (Piro et al. 1999). Evidence for an iron line was found. Although one might add to the list of bursts with well-observed late-time

light curves and additional spectral information in the X-ray band GRBs 990123, 000926, 010222, 011211, and 021004, the redshift of these bursts was $\gtrsim 1.5$, making it more or less hopeless to find an underlying SN component in the available data base (an upper limit for GRB 010222 is reported in Fig. 1b; Henden et al. 2004, in preparation). While it is very interesting that neither for GRB 020405 (Mirabal et al. 2003) nor for GRB 030329 (Tiengo et al. 2003) lines have been found in X-ray spectra of their afterglows, at present we cannot confirm a possible anti-correlation between the occurrence of X-ray lines and the appearance of SN light in GRB afterglows. This important issue remains to be investigated in the *Swift* era.

4.4. SN properties vs. afterglow parameters

Of particular interest is whether the burst and afterglow properties are to some degree related to the existence of an underlying SN component. For this reason we have investigated if the deduced SN luminosity is correlated with the corresponding energy release in the gamma-ray band (as given in Bloom, Frail, & Kulkarni 2003). No such correlation was found. We have also checked whether the afterglow parameters α_1 (pre-break decay slope), α_2 (post-break decay slope), and t_b (break time; Eq. A2) from those GRBs with detected SN component are different from those without such a component. Again, no correlation was apparent, even though one should keep in mind that the data base is still very small. For illustration, in Fig. 6 we display the relation between the afterglow parameters α_1 and α_2 for all GRB afterglows we have investigated. While there is a 'forbidden region' with $\alpha_2 \lesssim \alpha_1 + 3/4$ apparent in the data, with the border line representing the simplest jet model with no sideways expansion of the jet (Mészáros & Rees 1999), no bimodality in this distribution is visible. A tendency in our data that GRB afterglows with detected underlying SN component seem to prefer pre-break slopes $\alpha_1 > 1$ should not be overinterpreted, since the $\alpha_1 < 1$ sample includes several bursts with redshifts $z > 1.2$, where the photometric detection of an underlying SN component is difficult. On the other hand, at least one selection effect does occur here. If a bright SN component is apparent in the data then the parameter α_2 of the afterglow light curve is usually much more difficult to quantify, because the late-time evolution of the genuine afterglow is less well-defined. This problem is well seen in the afterglow light curve of GRB 011121 (e.g., Greiner et al. 2003) and GRB 030329 (e.g., Lipkin et al. 2004).

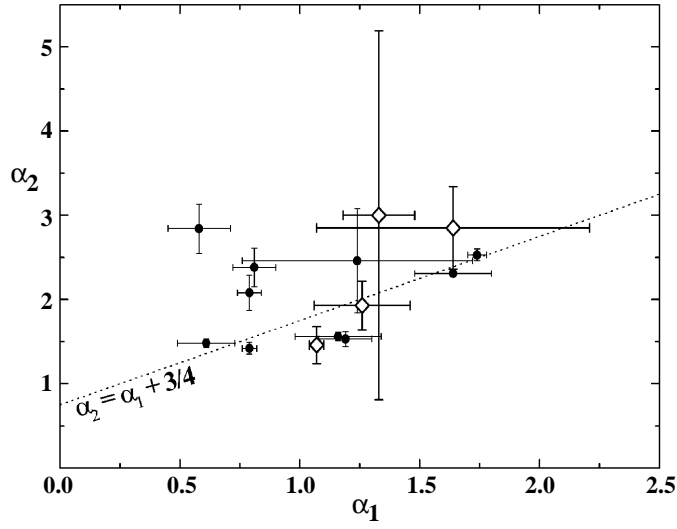


Fig. 6.— The correlation between the afterglow parameters α_1 and α_2 (Eq. A2) for all afterglows with a break in their light curves. The dotted line is the theoretical prediction in the simplest model ($\Theta_{\text{jet}}=\text{const}$; Mészáros & Rees 1999). Open diamonds indicate the four afterglows with a break and a detected underlying late-time bump, i.e., a SN component (GRB 980703, 011121, 020405, 021211). Note that several afterglows listed in Table 1 showed no evidence for a break in their light curves, so that they are not included in this figure.

5. Summary

In an attempt to study the GRB-SN association, we have re-analyzed in a systematic way all GRB afterglow data published by the end of 2002. We have found that in nine cases evidence for extra light at late times is apparent in the optical afterglow light curves. In most cases this is seen in more than one photometric band. This extra light can be modeled well as supernova light peaking $(1+z)15\ldots 20$ days after a burst. Our main finding is that all GRB afterglows with redshift $z \lesssim 0.7$ showed evidence for extra light at later times. For larger redshifts the data base is usually not of sufficient quality, or the SN is simply too faint, in order to search for such a feature in the late-time afterglow light curve.

The cut off date of our sample (end of 2002) was chosen to ensure that all GRBs had published follow-up observations. Since that date, five new afterglows with redshifts have been established. All but one were at redshifts above 0.7, and again for the only nearby event (GRB 030329) a supernova component was established. This is consistent with the statistical inferences from the sample of earlier long-duration GRBs and leads us to conclude that the current world sample of GRB afterglow measurements provides strong statistical support for the link between (long-duration) GRBs and the final stages of massive star evolution. This conclusion basically agrees with earlier reports by Dado, Dar, & de Rújula (2002a; and references therein), and is essentially independent of the underlying GRB model. While so far only one event (GRB 030329) allowed a direct spectroscopic confirmation of this link, the larger photometric sample discussed here supports this idea by statistical means.

Based on our sample of nine GRB-supernovae we have performed a first statistical approach to get insight on the characteristic luminosities of this type of supernovae. We have found strong evidence that SN 1998bw is at the bright end of the GRB-SN luminosity distribution, with the latter matching well into what is known so far about the luminosities of the brightest members of other types of core collapse supernovae (Richardson et al. 2002). While GRB-SNe are not standard candles, their peak luminosities are comparable to those of Type Ia. In fact, within the context of the SN interpretation of the late-time bumps in afterglows, our results demonstrate once more that the first years in GRB research have already provided a first sample of high- z core collapse SNe up to a redshift of 1. This sample might grow rapidly in the near future if indeed all long-duration GRBs tell us when and where in the universe a massive star explodes.

Some caution is of course required. First, there is some bias in the sample of bursts with detected optical afterglows. None of the bursts with a detected SN bump was classified as an X-ray rich burst or an X-ray flash, and for none of them were X-ray lines reported in the literature. In other words, it is still possible that bursts with SN bumps do not belong to these classes of events (but see Fynbo et al. 2004). Second, for most of the bursts discussed

here evidence for a SN bump is based on a very small number of data points around the SN peak time (say, between 10 to 40 days after the burst), with the most critical cases being GRB 991208 and 010921. However, we see no reason why we should disregard these events.

In their discovery paper, Klebesadel, Strong, & Olson (1973) noted that a potential relation of GRBs to supernovae might still be an option to explain this new phenomenon. While the model they refer to (Colgate 1968) does not describe what is today believed to be the underlying GRB mechanism, historically it is nevertheless remarkable that the first paper ever about GRBs might have given the right hint on the underlying source population, followed by many years of trial and error.

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A. Numerical approach

A.1. The light curve of the optical transient

We model the light curve of the optical transient (OT) following a GRB as a composite of afterglow (AG) light, supernova (SN) light, and constant light from the underlying host galaxy. The flux density, F_ν , at a frequency ν is then given by

$$F_\nu^{\text{OT}}(t) = F_\nu^{\text{AG}}(t) + k F_\nu^{\text{SN}}(t/s) + F_\nu^{\text{host}}. \quad (\text{A1})$$

Here, the parameter k describes the observed brightness ratio (in the host frame) between the GRB-supernova, and the SN template (SN 1998bw) in the considered photometric band (in the observer frame). We allowed k to be different in every photometric band, but within a band independent of frequency. The parameter s is a stretch factor with respect to the used template. We have also explored the consequences of a shift in time between the onset of the burst and the onset of the supernova explosion, as implied by certain theoretical models (Vietri & Stella 2000). Then, in Eq. (A1) $F_\nu^{\text{SN}}(t/s)$ was replaced by $F_\nu^{\text{SN}}(t + \tau)$. Here, $\tau = 0$

refers to GRB 980425/SN 1998bw (Iwamoto et al. 1998). If $\tau < 0$ the SN preceded the onset of the GRB.

We describe the afterglow light curve by a broken power-law (Beuermann et al. 1999; Rhoads & Fruchter 2001),

$$F_{\nu}^{\text{AG}}(t) = \text{const} [(t/t_b)^{\alpha_1 n} + (t/t_b)^{\alpha_2 n}]^{-1/n}, \quad (\text{A2})$$

with $\text{const} = 2^{1/n} 10^{-0.4m(t_b)}$. Here t is the time after the burst (in the observer frame), α_1 is the pre-break decay slope of the afterglow light curve, α_2 is the post-break decay slope, and t_b is the break time. The parameter n characterizes the sharpness of the break; a larger n implies a sharper break. In most cases the parameter n (Eq. A2) had to be fixed, otherwise the iteration did not converge. The reason was the usually too small number of data points around the break time. In these cases we set $n = 10$, producing a relatively sharp break in the light curve. However, this procedure did not strongly affect the deduced supernova parameters k and s (Eq. A1).

In the observer frame the flux density of the time-dependent supernova light is given by (cf. Dado et al. 2002a)

$$F_{\nu}^{\text{SN}}(t) = \frac{1 + z_{\text{SN}}}{1 + z_{\text{bw}}} \frac{d_{L,\text{bw}}^2}{d_{L,\text{SN}}^2} F_{\text{bw}}\left(\nu \frac{1 + z_{\text{SN}}}{1 + z_{\text{bw}}}; t \frac{1 + z_{\text{bw}}}{1 + z_{\text{SN}}}\right). \quad (\text{A3})$$

Here 'SN' stands for the GRB supernova under consideration, and 'bw' represents SN 1998bw ($z=0.0085$; Tinney, Stathakis, & Cannon 1998). We calculated the luminosity distance, d_L , assuming a flat universe with matter density $\Omega_M = 0.3$, cosmological constant $\Omega_{\Lambda} = 0.7$, and Hubble-constant $H_0 = 65 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

We always fitted photometric magnitudes. After manipulating Eqs. (A1, A2), the apparent magnitude of the OT in a given photometric band is given as

$$m_{\text{OT}}(t) = -2.5 \log\{10^{-0.4m_c}[(t/t_b)^{\alpha_1 n} + (t/t_b)^{\alpha_2 n}]^{-1/n} + k 10^{-0.4m_{\text{SN}}(t/s)} + 10^{-0.4m_{\text{host}}}\}. \quad (\text{A4})$$

Again, t/s was replaced by $t + \tau$ when we allow for a delay between SN and GRB. Equation (A4) has eight free parameters: $\alpha_1, \alpha_2, n, t_b, k, s, m_{\text{host}}$, and the constant m_c , which absorbs the constant of Eq. (A2) and corresponds to the magnitude of the fitted light curve for the case $n = \infty$ at the break time t_b . If there is no break in the light curve, then Eq. (A4) reduces to

$$m_{\text{OT}}(t) = -2.5 \log\{10^{-0.4m_1}t^{\alpha} + k 10^{-0.4m_{\text{SN}}(t/s)} + 10^{-0.4m_{\text{host}}}\}, \quad (\text{A5})$$

where m_1 is the brightness of the afterglow at $t = 1$ day after the burst (if t is measured in days).

A.2. Redshifting the SN 1998bw light curves

Equation (A1) requires as an input the function $F_\nu^{\text{SN}}(t)$ for arbitrarily frequencies in the optical bands. Spectra from SN 1998bw are available in the literature, but the time coverage of published broad-band photometry is much better. Therefore, we constructed $F_\nu^{\text{SN}}(t)$ based on published $UBVRI$ light curves (Galama et al. 1998), assuming that we can smoothly interpolate between adjacent photometric bands. Thereby, we have taken into account that various broad-band features are inherent to the spectral energy distribution of SN 1998bw that develop with time (e.g., Patat et al. 2001; Stathakis et al. 2000). Therefore, for different photometric bands SN 1998bw light curves peak at different times t_ν^{max} , at different flux densities, $F_\nu^{\text{max}} = F_\nu(t_\nu^{\text{max}})$, and have different shapes.² In the following, we demonstrate our numerical approach using the U and B bands as an example (Fig. 7).

Let ν_U be the central frequency of the U band, ν_B be the central frequency of the B band, and ε be defined as $0 \leq \varepsilon \leq 1$. Let ν' be the frequency for which the light curve $F_\nu(t)$ is required, then the relation $\nu' = \nu_U + \varepsilon(\nu_B - \nu_U)$ defines the value of ε . Assuming a smooth behavior of $F_\nu(t)$ between the U band and the B band, for the frequency-dependent peak flux of the light curve at the frequency ν' , we assume

$$\log F_{\nu'}^{\text{max}} = \log F_U^{\text{max}} + \varepsilon (\log F_B^{\text{max}} - \log F_U^{\text{max}}). \quad (\text{A6})$$

Similarly, for the frequency-dependent peak time of the supernova light curve at a fixed frequency, ν' , we write

$$t_{\nu'}^{\text{max}} = t_U^{\text{max}} + \varepsilon (t_B^{\text{max}} - t_U^{\text{max}}). \quad (\text{A7})$$

Finally, in order to model the frequency-dependent shape of the SN 1998bw light curves, we normalize them to their peak flux and peak time. Then, at a given frequency we have $F_\nu(t) = \eta_\nu F_\nu^{\text{max}}$, where η is a function of the ratio t/t_ν^{max} and $0 \leq \eta \leq 1$. Correspondingly, our ansatz for the shape function $\eta_{\nu'}$ for a redshifted SN 1998bw is

$$\log \eta_{\nu'}(x) = \log \eta_U(x) + \varepsilon [\log \eta_B(x) - \log \eta_U(x)], \quad (\text{A8})$$

where $x = t_{\text{host}}/t_{\nu'}^{\text{max}}$ and

$$t_{\text{host}} = t \frac{1 + z_{\text{bw}}}{1 + z_{\text{SN}}} \quad (\text{A9})$$

is measured in the host frame (symbols follow Eq. A3).

Once an ensemble of functions $F_\nu(t)$ has been calculated, the apparent magnitude of the redshifted SN 1998bw in a given photometric band (Eqs. A4, A5) is obtained by integrating over the flux density per unit wavelength ($F_\lambda(t) = -\nu^2/c F_\nu(t)$), multiplied by the

²For reasons of clarity, in this section we omit the index 'SN' at F_ν ; all flux densities refer to SN 1998bw.

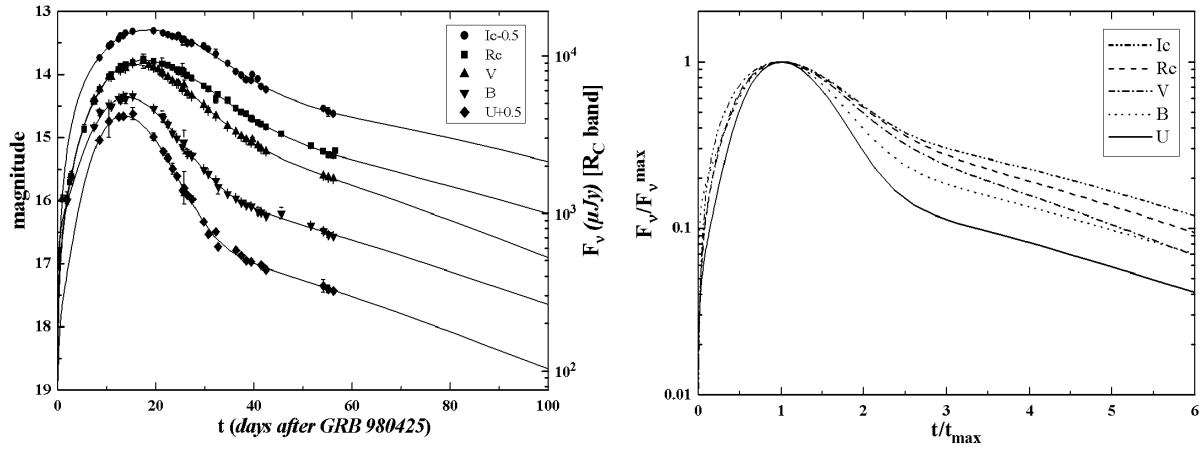


Fig. 7.— *Left:* The $UBVRI$ light curves of SN 1998bw according to the data provided by Galama et al. (1998). The fits (drawn through lines) are based on a purely empirical equation which fits a supernova light curve very well and is not physical. It extrapolates also beyond 60 days. Note that the light curves differ in peak flux, peak time, and in shape. In order to predict the light curves of a redshifted SN 1998bw one has to construct light curves for any frequency in between the characteristic frequencies of the $UBVRI$ bands. *Right:* The broad-band light curves of SN 1998bw normalized to their peak maxima and peak times. Now they differ only in their shapes.

corresponding filter response function S_λ . For S_λ we used the transmission curves for Bessel filters provided on the internet pages of the *European Southern Observatory* for VLT-FORS1 with reference to Bessel (1979). For the transformation between photometric magnitudes and flux densities we used the calibration constants provided by Fukugita, Shimasaku, & Ichikawa (1995; their table 9) and Zombeck (1990). The so calculated broad-band light curves of a redshifted SN 1998bw were then used as an input function for Eqs. (A4, A5).

The results of our numerical procedure were compared with corresponding results published by Dado et al. (2002a) and Bloom et al. (2002), and we found close agreement. We used our procedure to correctly *predict* the color evolution of GRB-SN 030329 (Zeh, Klose, & Greiner 2003), and have performed a very good numerical fit for the light curves of GRB-SN 011121 (Greiner et al. 2003b). The limit of our procedure is given by the chosen photometric band in combination with the redshift of the burster. Once we can no longer interpolate in between the *UBVRI* bands, but have to extrapolate into the UV domain (cf. Bloom et al. 1999), results become less accurate.

B. Notes on individual bursts with detected supernova bump

GRB 970228: The light curves were constructed based on the data compiled by Galama et al. (2000). The R_c band light curve shows evidence for extra light appearing ~ 2 weeks after the burst, which can be attributed to an underlying SN 1998bw component at the redshift of the burster (Galama et al. 2000; Reichart 1999). There is no evidence for a break, but its presence cannot be excluded due to the rather sparse data set. In the I_c band the SN bump is most visible, but the light curve suffers from a lack of early-time data. In the V band the significance for extra light is even lower, again due to the lack of observational data.

GRB 980703: The search of an SN component in the afterglow of GRB 980703 is affected by the relatively bright host. Most data were taken from Bloom et al. (1998), Castro-Tirado et al. (1999), Holland et al. (2001), and Vreeswijk et al. (1999). Evidence for a late-time bump is rather weak.

GRB 990712: This burst had a relatively bright host galaxy hampering the long-term study of its afterglow. We used the data presented by Fruchter et al. (2000a), Hjorth et al. (2000), and Sahu et al. (2000) to analyze the light curves. Although the R_c band light curve is well-sampled, the bright host may have hidden the detectability of a break at later times. Our numerical procedure finds evidence for extra-light, confirming the finding by Björnsson et al. (2001).

GRB 991208: We used the compilation of data by Castro-Tirado et al. (2001), with additional data from Dodonov et al. (1999), Halpern et al. (1999), Garnavich et al. (1999), and Fruchter et al. (2000b), including our late-time observation of the host in early 2003 to analyze the light curves. The R_c band data can be fitted with or without the inclusion of a break. In the former case the break is mainly due to a single data point at $t \sim 7$ days. Most likely, the afterglow was discovered after a break had already occurred in the light curve. The afterglow parameters were determined in the R_c band. These parameters fit well in the V band. Unfortunately, in the I_c band no data were obtained during the time of the SN bump.

GRB 000911: This was a long-lasting burst with a duration of ~ 500 seconds (Hurley et al. 2000; Price et al. 2002b). The optical afterglow was in detail observed by Price et al. (2002b) and Lazzati et al. (2001). We confirm the finding by Lazzati et al. that the published data show evidence for a bump in VRI at later times.

GRB 010921: This burst occurred in a rather crowded stellar field and had a relatively large error box (Hurley et al. 2001), which hampered the early detection of its afterglow (Price et al. 2001). The light curve is therefore not well-sampled. Using the data published by Park et al. (2002) and Price et al. (2002a) combined with our late-time observations of the host, our numerical procedure finds evidence for extra light with its peak time ~ 2 weeks after the burst. This result is obtained when we adopted a single power-law decay, where the decay slope α was deduced from the r' band light curve. Our procedure finds a SN component with $k = 0.68 \pm 0.48$ and $s = 0.68 \pm 0.28$. Within the given uncertainties this is not in conflict with the upper limit reported by Price et al. (2003).

GRB 011121: This was the nearest known burst at the time of its discovery (excluding GRB 980425/SN 1998bw). It showed clear evidence for an underlying SN component in several photometric bands (Bloom et al. 2002; Dado, Dar, & de Rújula 2002b; Garnavich et al. 2003; Greiner et al. 2003b). In our fit we included late-time *Hubble Space Telescope* data (Bloom et al. 2002). A break is apparent in the light curve at $t \sim 1$ day (see Greiner et al. 2003b; note that in Greiner et al. we assumed a Galactic extinction towards SN 1998bw of 0 mag).

GRB 020405: This is the second burst with known redshift and a well-observed bump in its late-time afterglow. We used host-subtracted data provided by N. Masetti (private communication) to analyze the light curves. Extra light apparent in the late-time light curve can be attributed to an underlying SN component, as already noted by Masetti et al. (2003). Our procedure also detects a break in the light curves at $t \sim 2$ days.

GRB 021211: For the fit we included data published by Della Valle et al. (2003), Fox

et al. (2003), Li et al. (2003), and Pandey et al. (2003). A weak bump is apparent at late times, which is most likely due to an underlying SN component given its (weak) spectral confirmation (Della Valle et al. 2003).

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Table 1: The input sample of GRB afterglows^a

GRB	z	GRB	z	GRB	z
970228	0.695	991208	0.706	010921	0.450
970508	0.835	991216	1.02	011121	0.362
971214	3.42	000301C	2.04	011211	2.140
980703	0.966	000418	1.118	020405	0.69
990123	1.600	000911	1.058	020813	1.25
990510	1.619	000926	2.066	021004	2.3
990712	0.434	010222	1.477	021211	1.01

^aRedshifts were taken from the literature.

Table 2: Best-fit parameters for the SN component found in GRB afterglows^a

GRB	z	band	λ_{host}	k	s	$\chi^2_{\text{d.o.f.}}$	k if $s=1$	$\chi^2_{\text{d.o.f.}}$	χ^2_{noSN}	data
970228	0.695	I_c	476	0.66 ± 0.27	0.01	2.16	4
		R_c	389	0.40 ± 0.24	1.46 ± 0.80	0.70	0.33 ± 0.30	0.71	0.77	10
		V	325	0.25 ± 0.50	0.06	0.15	4
980703	0.966	I_c	410	1.59	14
		R_c	335	1.66 ± 1.22	0.78	0.79	19
		V	280	1.50	7
990712	0.434	I_c	562	1.00 ± 0.38	0.56 ± 0.10	0.55	1.67	6
		R_c	459	0.48 ± 0.10	0.89 ± 0.10	1.00	0.43 ± 0.08	1.01	2.25	23
		V	384	0.37 ± 0.44	0.71 ± 0.36	2.30	0.29 ± 0.18	1.62	1.99	16
991208	0.706	I_c	472	1.82	13
		R_c	386	0.90 ± 0.35	1.12 ± 0.28	1.64	1.02 ± 0.32	1.56	2.52	20
		V	323	1.16 ± 0.19	1.86 ± 0.10	0.45	0.93 ± 0.30	1.04	2.26	11
000911	1.058	I_c	392	0.39 ± 0.37	1.06 ± 0.50	1.83	0.40 ± 0.29	1.30	1.61	7
		R_c	320	0.87 ± 0.39	1.49 ± 0.33	0.75	0.51 ± 0.43	1.14	1.22	8
		V	267	0.43 ± 1.24	1.25	1.42	6
010921	0.450	I_c	556	0.40 ± 1.67	0.50	0.28	4
		R_c	454	0.68 ± 0.48	0.68 ± 0.28	0.42	0.43 ± 0.10	0.78	2.74	6
		V	380	2
011121	0.360	I_c	632	0
		R_c	484	0.79 ± 0.06	0.85 ± 0.06	0.92	0.74 ± 0.05	1.32	>20	13
		V	405	0.86 ± 0.09	0.81 ± 0.06	2.13	0.83 ± 0.05	3.26	>20	10
020405	0.695	I_c	476	0.76 ± 0.17	0.80 ± 0.17	5.29	0.71 ± 0.10	5.68	>20	10
		R_c	389	0.74 ± 0.17	0.98 ± 0.17	5.26	0.72 ± 0.11	4.86	>20	18
		V	325	0.69 ± 0.22	0.74 ± 0.13	6.79	0.53 ± 0.16	6.91	>20	14
021211	1.006	I_c	428	0
		R_c	328	0.97 ± 0.87	0.74 ± 0.23	2.68	0.52 ± 0.34	2.65	2.79	35
		V	274	0

^aColumns: (1) and (2): GRB and redshift; (3) photometric band, in which the light curve was fitted; (4) central wavelength of the photometric band in the host frame in units of nm, adopting for V, R_c, I_c wavelengths of 550, 659, and 806 nm, respectively; (5) peak luminosity of the fitted SN component in the corresponding wavelength band (observer frame) in units of SN 1998bw, after correction for Galactic extinction; (6) stretch factor s (Eq. A1); (7) goodness of fit per degree of freedom; (8) and (9) the same as (5) and (6) for $s = 1$; (10) goodness of fit per degree of freedom assuming that there is no underlying SN component; (11) total number of data points used for the fit. Note that the low $\chi^2/\text{d.o.f.}$ for GRB 970228 is due to the small number of data points.